MCE415
Heat and Mass Transfer

Lecture 04: 02/10/2017

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Class: Monday (1 – 3 pm)
Venue: B13

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Natural Convection Heat Transfer

- Physical Mechanism of Natural Convection
- Equation of Motion and Grashof Number
- Natural Convection Over Surfaces
- Rayleigh Number

Recommended textbook

- Fundamentals of Thermal-Fluid Sciences by Cengel Y.A., Turner R.H., & Cimbala J.M. 3rd edition

HANDS-ON ACTIVITY

- A person extends his uncovered arms into the windy air outside at 10 °C and 30 km/h in order to feel nature closely. Initially, the skin temperature of the arm is 30 °C. Treating the arm as a 0.6 m long and 7.5 cm-in-diameter cylinder, determine the rate of heat loss from the arm.

- Reconsider the question above. Using EES (or other) software, investigate the effects of air temperature and wind velocity on the rate of heat loss from the arm. Let the air temperature vary from -5 °C to 25 °C and the wind velocity from 15 km/h to 60 km/h. Plot the rate of heat loss as a function of air temperature and of wind velocity, and discuss the results.

PHYSICAL MECHANISM OF NATURAL CONVECTION

- Natural convection occurs when fluid motion is initiated by natural means such as *buoyancy*. The velocity is usually less than 1 m/s hence low convection coefficient which makes it hardly noticeable.
- Natural convection heat transfer is experienced during cooling of electronic devices eg TVs, VCRs, power transistors; heat transfer in refrigeration coils; and in human and animal bodies.

Heat transfer from a body is usually a combination of *natural convection to the air* and *radiation to the surrounding surfaces*.

- The continual replacement of the heated air around the egg by the cooler air nearby is caused by motion called a ***natural convection current***.
- The heat transfer that is enhanced as a result of this natural convection current is called ***natural convection heat transfer***.

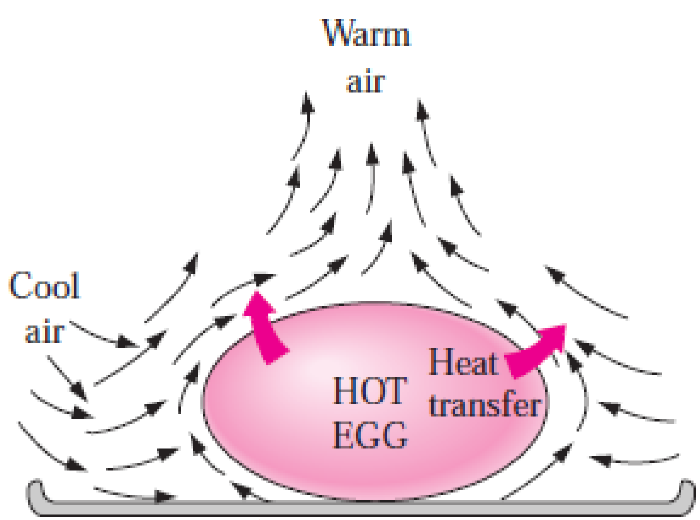


Fig 1: The cooling of a boiled egg in a cooler environment by natural convection

PHYSICAL MECHANISM OF NATURAL CONVECTION

- In a *gravitational field*, a net force pushes upward a lighter fluid when in contact with a heavier fluid. The upward force of the fluid that pushes against a body immersed in it is called *buoyancy force*.
- From Archimedes principle, a net force exist between a body and the fluid in which it is immersed and it is proportional to the difference in the densities of the body and the fluid as shown in Eq 1

$$\begin{aligned} F_{net} &= W - F_{buoyancy} \\ &= \rho_{body}\, g V_{body} - \rho_{fluid}\, g V_{body} \\ &= (\rho_{body} - \rho_{fluid})\, g V_{body} \end{aligned} \qquad 1$$

- The exchange of heat by natural convection between a cold (or hot) surface and the surrounding air occurs due to the *buoyancy effect*. In its absence it is purely by conduction.

NOTE: There is no gravity in space therefore natural convection heat transfer cannot take place in a spacecraft.

PHYSICAL MECHANISM OF NATURAL CONVECTION

- Temperature is a primary variable in heat transfer studies and the property ***β***, ***volume expansion coefficient***, helps to relate $\Delta\rho$ and ΔT.
- ***Volume expansion coefficient***, ***β***, is defined as the variation in density of a fluid with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K}) \qquad 2$$

- Approximating the differentials as difference yields Eq. 3

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad \text{(at constant } P) \qquad 3$$

Where ρ_∞ = density , T_∞ = temperature of the quiescent fluid away from the surface.

- For an ideal has β becomes $\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K})$ 4

The *larger the temperature difference* between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger the buoyancy force*, the *stronger the natural convection currents*, thus *the higher the heat transfer rate*

PHYSICAL MECHANISM OF NATURAL CONVECTION

- The magnitude of heat transfer rate is determined by the flow rate such that the higher the flow rate the larger the heat transfer rate.
- There is however a competing effect between *buoyancy* and *surface friction* that establishes the fluid flow rate.
- Friction forces increase as more solid surfaces are introduced; seriously disrupting the flow rate and heat transfer . For this reason closely packed fins are unsuitable for natural convection cooling.
- Most heat transfer correlations in natural convection are based on experimental measurements and the instrument often used is the *Mach–Zehnder interferometer*, which gives a plot of isotherms in the fluid in the vicinity of a surface

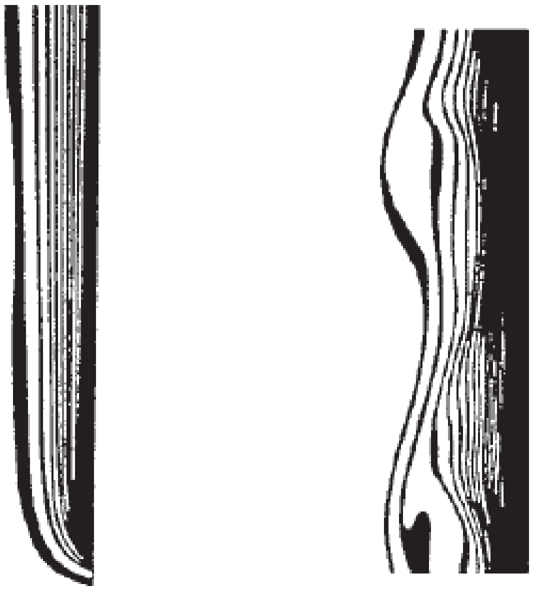


Fig 2: Isotherms in natural convection over a hot plate in air

EQUATION MOTION AND GRASHOF NUMBER

- The equation of motion for natural convection in laminar boundary layer is based on conservation of energy and mass as well as conservation momentum, which incorporates the buoyance effect.
- The following assumptions are made
  - Steady state, 2D, laminar flow
  - Newtonian fluid with constant properties
  - However, $\Delta\rho$ rather density is considered
- Since it is the $\Delta\rho$ between the inside and outside of the boundary layer that initiates buoyancy force and sustains the flow. (This is known as *Boussinesq approximation*)
- The x- and y-components of the velocity within the boundary layer are $u = u(x, y)$ and $v = v(x, y)$

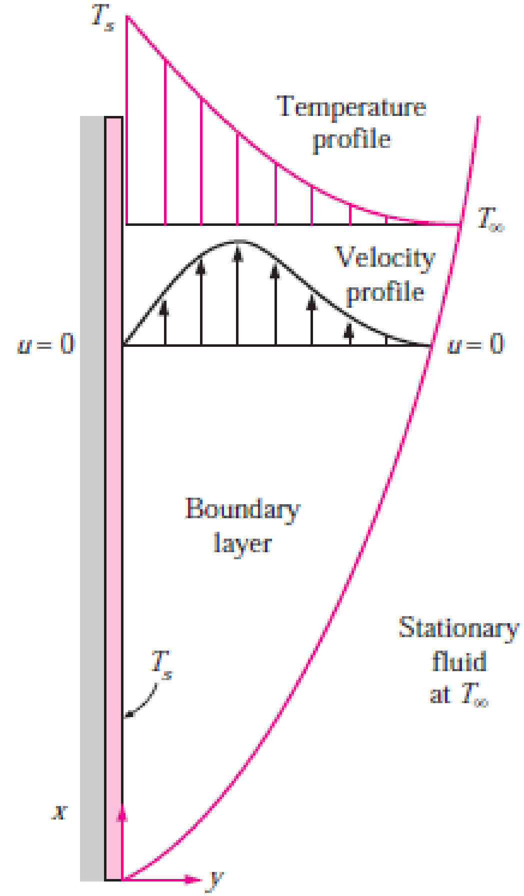


Fig 3: Typical velocity and temp profiles for natural convection flow over a hot vertical plate temp T_s inserted in a fluid of temp T_∞

EQUATION MOTION AND GRASHOF NUMBER

- The temperature and velocity profiles are a shown if Fig 3.
- Boundary layer thickness increases in the direction of flow just like in forced convection
- However the velocity is zero both at the plate surface and the outer edge of the boundary layer.

WHY?

- The velocity profile increases reaches a maximum value and decreases, while the temperature profile drops from a peak (plate surface temperature) to the air temperature at a point sufficiently far away from the surface.

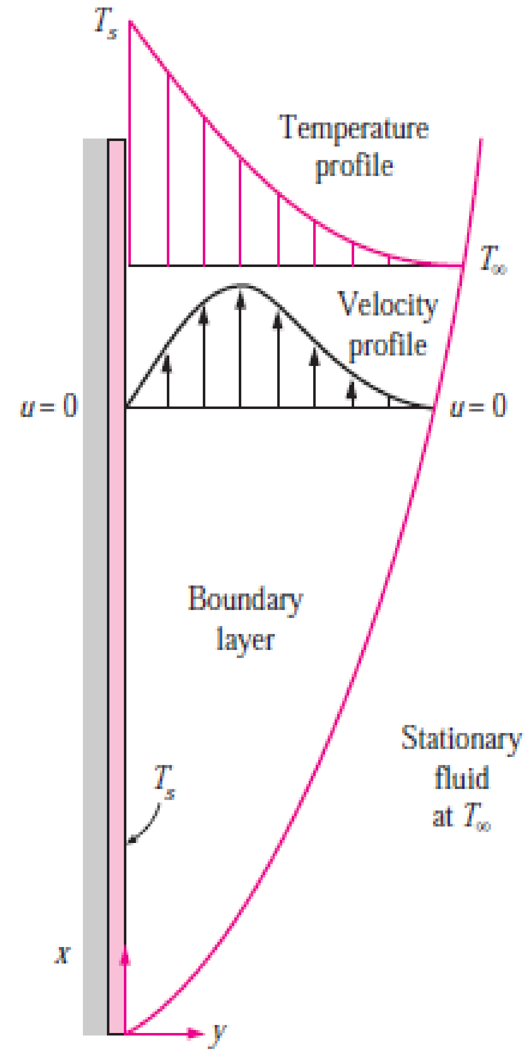


Fig 3: Typical velocity and temp profiles for natural convection flow over a hot vertical plate temp T_s inserted in a fluid of temp T_∞

EQUATION MOTION AND GRASHOF NUMBER

- Considering a differential volume, applying Newton's second law of motion and following a few manipulations, the momentum equation yields Eq. 5

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \qquad 5$$

- Eq 5 governs the fluid motion in the boundary layer due to the effects of buoyancy.
- The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities as shown below:

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{V} \qquad \upsilon^* = \frac{\upsilon}{V} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty} \qquad 6$$

- Substituting in to Eq 5 and simplifying yields Eq. 7

$$u^*\frac{\partial u^*}{\partial x^*} + \upsilon^*\frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right]\frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad 7$$

$$u^* \frac{\partial u^*}{\partial x^*} + \upsilon^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}} \qquad 7$$

- The dimensionless parameter in the bracket represents the natural convection effects and it is termed Grashof Number

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2} \qquad 8$$

where

g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
υ = kinematic viscosity of the fluid, m^2/s

- The Grashof Number is the criterion to determine when transition from laminar to turbulent flow occurs in natural convection

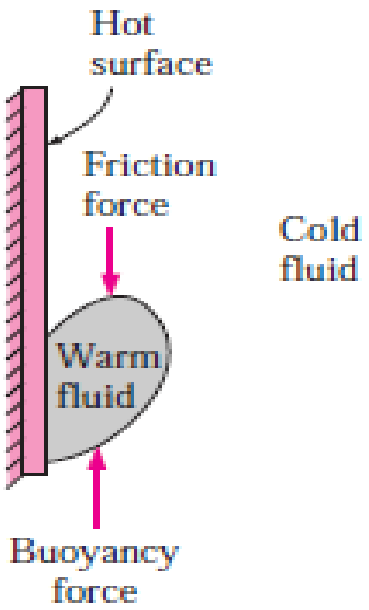


Fig 4: The Grashof number Gr is a measure of the relative magnitudes of the buoyancy force and the opposing viscous force acting on the fluid

EQUATION MOTION AND GRASHOF NUMBER

- The critical Grashof Number is often set at 10^9, therefore a natural convection flow regime over a vertical plate below this value is laminar while above it is considered a turbulent flow.

> NOTE: When a surface is subjected external flow, the problem involves both forced and natural convection.

- The relative importance of the two conditions is determined by this term ${Gr_L}/{Re_L^2}$
- Natural convection effects are negligible if ${Gr_L}/{Re_L^2} \ll 1$
- Forced convection effects are negligible if ${Gr_L}/{Re_L^2} \gg 1$, while the effects of free convection dominates.
- Both effects are important and must be considered if ${Gr_L}/{Re_L^2} \approx 1$.

NATURAL CONVECTION OVER SURFACES

- Natural convection depends on the
  i. geometry of the surface
  ii. orientation of the object
  iii. temperature variation of the surface, and
  iv. thermophysical properties of the fluid involved.
- It is impracticable to have simple analytical relations from the governing equations of motion and energy for natural convection therefore they are often determined experimentally
- However those that exist for simple geometries are usually of the form

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L \text{Pr})^n = C\,\text{Ra}_L^n \qquad 9$$

- Where Ra_L is the ***Rayleigh number***, a product Grashof and Prandtl numbers (Eq 10)

$$\text{Ra}_L = \text{Gr}_L \text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr} \qquad 10$$

- The values of the constants C and n depend on the surface ***geometry*** and ***flow regime***, which is defined by the range of Rayleigh number
- Usually $n = {}^1/_4$ for laminar flow

  $n = {}^1/_3$ for turbulent flow

  $C < 1$
- All fluid properties are evaluated at film temperature

$$T_f = {}^{(T_s + T_\infty)}/_2 \qquad 11$$

- Once the average *Nusselt number,* Nu, is known, the average heat transfer coefficient, *h*, can be evaluated and Newton's law of cooling can be used to obtain the heat transfer rate as in Eq. 12

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad \text{(W)} \qquad 12$$

- Simple relations for average Nu, characteristic length, and the range of Ra for various geometries are shown in a Table, together with the geometries sketches

NATURAL CONVECTION OVER SURFACES

VERTICAL PLATES ($T_s = constant$)

- The characteristic length is the plate height, L (the Table) and the most complex relation is recommended for use for $10^{-1} < Ra_L < 10^9$

VERTICAL PLATES ($\dot{q}_s = constant$)

- In this case heat transfer rate is, $\dot{Q}$ but T_s is unknown. Note that T_s increases with height along the plate.
- The isothermal plates relation is also applicable here, but the plate midpoint temp, $T_{L/2}$, will be used for T_s in the evaluating of film temp, Ra and the Nu.
- Note that

$$\mathrm{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \qquad 13$$

VERTICAL CYLINDERS

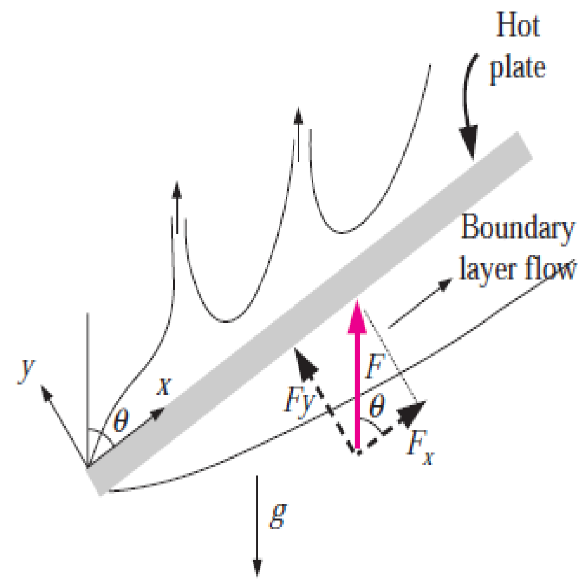
- A vertical cylinder can be treated as vertical plate if the condition below is met

$$D \geq \frac{35L}{\mathrm{Gr}_L^{1/4}} \qquad 14$$

NATURAL CONVECTION OVER SURFACES

INCLINED PLATES

- In case of an inclined plate as shown, the net force that drives the motion can be resolved into $F_y = \cos\theta$ and $F_x = \sin\theta$ normal to the plate.
- Consequently the force that drives the motion is reduced and convection currents are weaker, and the heat transfer rate is lower relative to a vertical plate



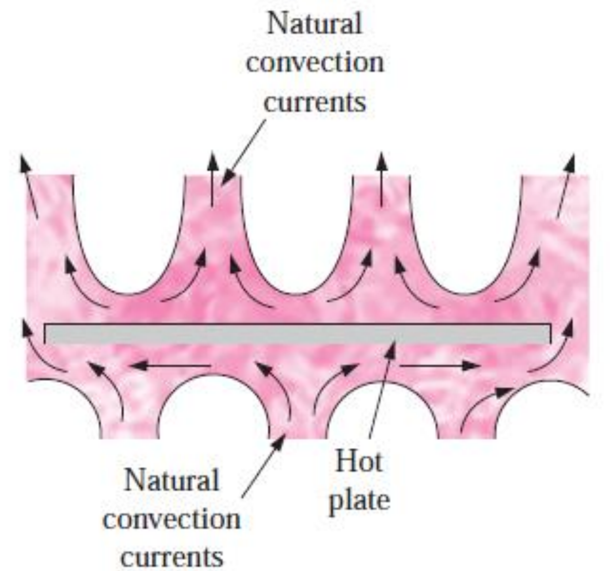
- Experiments confirm the above assertion for the lower surface but opposite for the upper surface. F_y component is responsible for this variation

NATURAL CONVECTION OVER SURFACES

HORIZONTAL PLATES

- The rate of heat transfer to or from a horizontal surface depends on whether the surface is facing upward or downward.
- The characteristic length for horizontal surfaces is calculated from

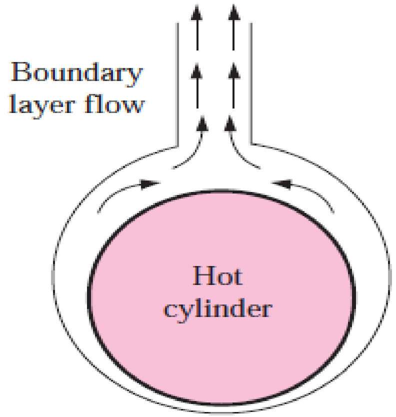
$$L_c = \frac{A_s}{p} \qquad 15$$



- Where A_s is surface area and p is perimeter. Note that L_c is $D/4$ for a horizontal circular surface of diameter D.C

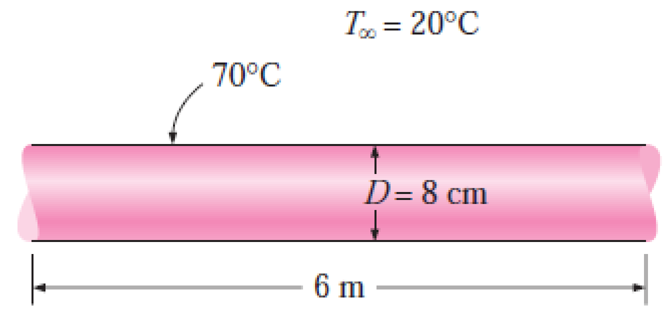
HORIZONTAL CYLINDERS AND SPHERES

- The boundary layer over a hot horizontal surface cylinder is as shown on the Fig and this governs the natural convection heat transfer.

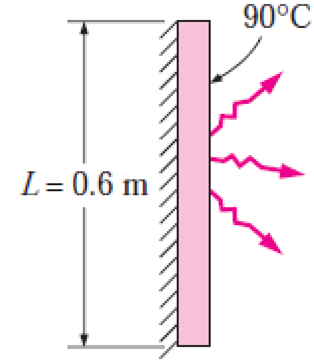


EXAMPLE

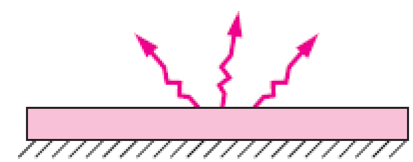
1. A 6-m-long section of an 8-cm-diameter horizontal hot-water pipe shown in the Fig passes through a large room whose temp is 20 °C. If the outer surface temp of the pipe is 70 °C, determine the rate of heat loss from the pipe by natural convection.



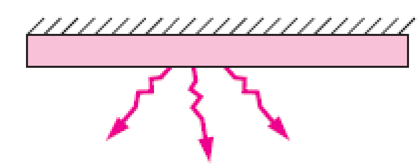
2. Consider a 0.6-m × 0.6-m thin square plate in a room at 30 °C. One side of the plate is maintained at a temp of 90 °C, while the other side is insulated, as shown in the Fig. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.



(*a*) Vertical

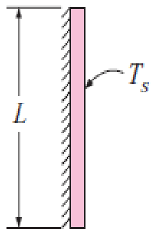
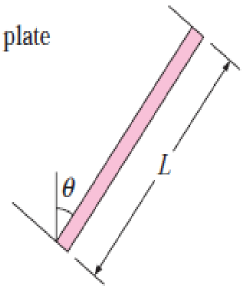
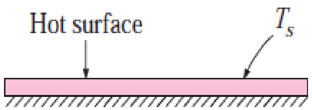


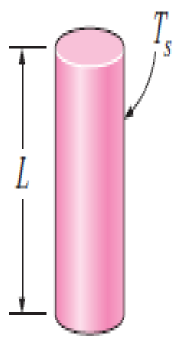
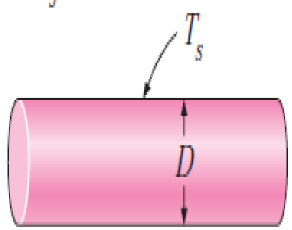
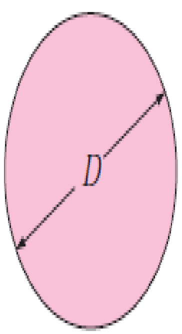
(*b*) Hot surface facing up



(*c*) Hot surface facing down

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{20}–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (20–19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (20–20) $\text{Nu} = \left\{0.825 + \dfrac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (20–21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (20–22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (20–23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (20–24)

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \ge \dfrac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \le 10^{12}$	$Nu = \left\{0.6 + \dfrac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (20–25)
Sphere 	D	$Ra_D \le 10^{11}$ ($Pr \ge 0.7$)	$Nu = 2 + \dfrac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (20–26)